General Solutions for a Transverse to Longitudinal Emittance Exchange Beamline and Cavity Gradient Sensitivity

Raymond P. Fliller III

Revision 11/27/07

Abstract

The goal of this paper is to expand upon the development of an emittance echange beamline as developed in Cornacchia and Emma [1]. In particular, to clarify what types of beamlines are needed before and after a deflecting mode cavity to effect a perfect emittance exchange. General properties of the transfer matricies of these beamlines are listed and some specific cases are shown. The hope is that this will aid those designing such beamlines.

1 Introduction

A scheme to exchange the longitudinal emittance with one of the transverse emittances was proposed by Cornacchia and Emma in 2002 [1]. In this paper the authors show many of the properties of a beamline to generate such an exchange. Furthermore, a beamline is proposed based on a chicane with a deflecting mode cavity in the dispersive region. Such a beamline does not exchange the emittances without coupling between the two planes. A subsequent paper by Emma, *et. al.* shows the design of a beamline based on two identical doglegs with a deflecting mode cavity after the first dogleg [2]. This beamline will exchange the emittances without residual coupling.

It has been proposed to perform such an experiment at the A0 photinjector prior to it moving to NML. This will be the topic of Tim Koeth's thesis. In the design of a beamline for the experiment, it was decided that a standard chicane will not be used and some design will be chosen to completely exchange the emittances without coupling. The purpose of this paper is to expand upon the derivation of beamline properties in Ref. [1] and derive what are the the necessary properties of the subsections of the beamline to effect such an exchange.

2 General Properties

The matrix of an emittance exchange beamline takes the form of

$$M = M_{ac}M_{cav}M_{bc} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \tag{1}$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are 2×2 blocks of the matrix, M_{cav} is the cavity matrix, and M_{bc} and M_{ac} symplectic matricies for the before cavity and after cavity sections ofthe beamline respectively. It is assumed that there is no RF in either the before or after cavity sections of the emittance exchange beamline. The requirement of symplecticity means that M_{bc} and M_{ac} have the form of

$$M_{bc} = \begin{pmatrix} a & b & 0 & \eta \\ c & d & 0 & \eta' \\ c\eta - a\eta' & d\eta - b\eta' & 1 & \xi \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

$$M_{ac} = \begin{pmatrix} e & f & 0 & D \\ g & h & 0 & D' \\ gD - eD' & hD - fD' & 1 & \chi \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

.

The cavity matrix takes the form

$$M_{cav} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{pmatrix} \tag{4}$$

as given in reference [1]. For completeness, we note that the phase space variables are x, x', z, δ which are the transverse position and angle, the distance from the bunch center ($z > 0$ refers to the front of the bunch), and fractional momentum offset.

Using these matricies the matrix for the total emittance exchange is

$$M = \begin{pmatrix} cf(1+k\eta) + ae + ak(D - f\eta') & df(1+k\eta) + be + bk(D - f\eta') & fk & D + e\eta + f\eta' + k(D\eta + f\xi) \\ ch(1+k\eta) + ag + ak(D' - h\eta') & dh(1+k\eta) + bg + bk(D' - h\eta') & hk & D' + g\eta + h\eta' + k(D'\eta + h\xi) \\ \begin{bmatrix} c(hD - fD') + a(gD - eD' + k\chi) \\ +(c\eta - a\eta')\left[1 + k(hD - fD')\right] \end{bmatrix} & \begin{bmatrix} d(hD - fD') + b(gD - eD' + k\chi) \\ +(d\eta - b\eta')\left[1 + k(hD - fD')\right] \end{bmatrix} & 1 + k(hD - fD') & \begin{bmatrix} D(g\eta + h\eta') + D'(e\eta + f\eta') \\ +k\xi(hD - fD') + \xi + \chi(1 + k\eta) \end{bmatrix} \\ ak & bk & 0 & 1 + k\eta \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \tag{5}$$

This matrix automatically fulfills all of the relations outlined in Appendix A of Reference [1] since it is a symplectic matrix. According to Equations 28 of Reference [1] read

$$\epsilon_x^2 = |\mathbf{A}|^2 \epsilon_{x0}^2 + (1 - |\mathbf{A}|)^2 \epsilon_{z0}^2 + \lambda^2 \epsilon_{x0} \epsilon_{z0} \tag{6}$$

$$\epsilon_z^2 = (1 - |\mathbf{A}|)^2 \epsilon_{x0}^2 + |\mathbf{A}|^2 \epsilon_{z0}^2 + \lambda^2 \epsilon_{x0} \epsilon_{z0} \tag{7}$$

$$\tag{8}$$

where $\epsilon_{x,z}$ are the transverse and longitudinal emittances after the exchange, and $\epsilon_{x0,z0}$ are the emittances prior. These equations state that $\det\mathbf{A} = |\mathbf{A}| = 0$ is required for emittance exchange. The $\lambda^2\epsilon_{x0}\epsilon_{z0}$ term is not explicitly written out, but in the text below Equations 28 the authors state that "$\lambda^2 = 0$ if and only if all $\mathbf{A}_{ij} = 0$ or the trivial case of no coupling at all, where all $\mathbf{B}_{ij} = \mathbf{C}_{ij} = 0$." It is clear from Equation 5 that in the case where the cavity is off, $k = 0$, there is no coupling of the emittances. However, it is not clear that if $\mathbf{A}_{ij} = 0$ then $\lambda^2 = 0$.

From Equations 19 of Reference [1]

$$\lambda^2\epsilon_{x0}\epsilon_{z0} = \text{trace}\left\{(\mathbf{A}\sigma_{\mathbf{x}}\mathbf{A}^T)^a\mathbf{B}\sigma_{\mathbf{z}}\mathbf{B}^T\right\} = \text{trace}\left\{(\mathbf{C}\sigma_{\mathbf{x}}\mathbf{C}^T)^a\mathbf{D}\sigma_{\mathbf{z}}\mathbf{D}^T\right\} \tag{9}$$

where $\sigma_{\mathbf{x,z}}$ is the sigma matrix for the transverse and longitudinal phase space prior to the exchange , $\mathbf{X}^T$ denotes the matrix transpose and $\mathbf{X}^a$ denote the adjoint matrix. When written in terms of the $\mathbf{A}$ and $\mathbf{B}$ blocks of the M matrix this term becomes

$$\begin{aligned}
\lambda^2\epsilon_{x0}\epsilon_{z0} \;=\;& (\mathbf{A}_{21}\mathbf{B}_{11} - \mathbf{A}_{11}\mathbf{B}_{21})^2\sigma_x^2\sigma_z^2 + (\mathbf{A}_{21}\mathbf{B}_{12} - \mathbf{A}_{11}\mathbf{B}_{22})^2\sigma_x^2\sigma_{z'}^2 \\
&+(\mathbf{A}_{22}\mathbf{B}_{11} - \mathbf{A}_{12}\mathbf{B}_{21})^2\sigma_{x'}^2\sigma_z^2 + (\mathbf{A}_{22}\mathbf{B}_{12} - \mathbf{A}_{12}\mathbf{B}_{22})^2\sigma_{x'}^2\sigma_{z'}^2 \\
&+2(\mathbf{A}_{21}\mathbf{B}_{11} - \mathbf{A}_{11}\mathbf{B}_{21})(\mathbf{A}_{21}\mathbf{B}_{12} - \mathbf{A}_{11}\mathbf{B}_{22})\sigma_x^2\sigma_{zz'} \\
&+2(\mathbf{A}_{22}\mathbf{B}_{11} - \mathbf{A}_{12}\mathbf{B}_{21})(\mathbf{A}_{22}\mathbf{B}_{12} - \mathbf{A}_{12}\mathbf{B}_{22})\sigma_{x'}^2\sigma_{zz'} \\
&+2(\mathbf{A}_{21}\mathbf{B}_{11} - \mathbf{A}_{11}\mathbf{B}_{21})(\mathbf{A}_{22}\mathbf{B}_{11} - \mathbf{A}_{12}\mathbf{B}_{21})\sigma_{xx'}\sigma_z^2 \\
&+2(\mathbf{A}_{21}\mathbf{B}_{12} - \mathbf{A}_{11}\mathbf{B}_{22})(\mathbf{A}_{22}\mathbf{B}_{12} - \mathbf{A}_{12}\mathbf{B}_{22})\sigma_{xx'}\sigma_{z'}^2 \\
&+4\left[(\mathbf{A}_{21}\mathbf{B}_{12} - \mathbf{A}_{11}\mathbf{B}_{22})(\mathbf{A}_{22}\mathbf{B}_{11} - \mathbf{A}_{12}\mathbf{B}_{21})\right. \\
&\left.(\mathbf{A}_{21}\mathbf{B}_{11} - \mathbf{A}_{11}\mathbf{B}_{21})(\mathbf{A}_{22}\mathbf{B}_{12} - \mathbf{A}_{12}\mathbf{B}_{22})\right]\sigma_{xx'}\sigma_{zz'}
\end{aligned} \tag{10}$$

where it is clear that if all of $\mathbf{A}_{ij} = 0$ then $\lambda^2 = 0$. One can surmise from the form that if the above equation were written in terms of the $\mathbf{C}$ and $\mathbf{D}$ blocks of M then this term would be zero if $\mathbf{D}_{ij} = 0$. We will note at this point that the chicane designed in Reference [1] has only the $\sigma_{x'}^2\sigma_{z'}^2$ term.

$$\lambda^2\epsilon_{x0}\epsilon_{z0} = 4\eta^2\sigma_{x'}^2\sigma_{z'}^2 \tag{11}$$

3 SubBeamline Properties

Now that some of the general properties of the emittance exchange matrix have been fleshed out and the assumptions stated we want to answer the following question,"What is needed of the elements of M_{bc}, M_{cav}, M_{ac}, to effect a perfect emittance exchange?"

There are four equations, one for each element of the $\mathbf{A}$ block. At first glance there are 12 unknowns (a, b, c, d, e, f, g, h, k, η, η', D, D') However, the first 4 are related by the symplectic condition. This is true for the second 4 elements as well. This leaves 10 free parameters. One can solve the system of equations $\mathbf{A}_{ij} = 0$, and arrive at the following relations between the free parameters:

$$k = -\frac{1}{\eta} \tag{12a}$$

$$D = e\eta + f\eta' \tag{12b}$$

$$D' = g\eta + h\eta' \tag{12c}$$

Plugging these into the matrix for the emittance exchange, one gets

$$M = \begin{pmatrix} 0 & 0 & -\frac{f}{\eta} & e\eta + f\eta' - f\frac{\xi}{\eta} \\ 0 & 0 & -\frac{h}{\eta} & g\eta + h\eta' - h\frac{\xi}{\eta} \\ c\eta - a\eta' - a\frac{\chi}{\eta} & d\eta - b\eta' - b\frac{\chi}{\eta} & 0 & 0 \\ -\frac{a}{\eta} & -\frac{b}{\eta} & 0 & 0 \end{pmatrix}. \tag{13}$$

and the equation for the after cavity matrix becomes

$$M_{ac} = \begin{pmatrix} \frac{D - f\eta'}{\eta} & f & 0 & D \\ \frac{D'}{\eta} + \frac{\eta'}{D} + f\frac{D'\eta'}{D\eta} & \frac{fD'+\eta}{D} & 0 & D' \\ -\eta' & \eta & 1 & \chi \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{14}$$

Now we will note the following interesting points of the solution. Firstly, it is general involving only the properties of the cavity, symplectic transport lines without RF, and perfect emittance exchange. The second is that the solution of the cavity strength is only determined by the dispersion in the cavity. It is not obvious that it should not depend on other things, such as the slope of the dispersion through the cavity. Thirdly, the requirements of the after cavity transport only depend on the dispersion and its slope in the cavity. It does not depend on the other elements of the transport matrix prior to the cavity. Fourth there are no requirements for the elements of M_{bc} other than it generate the required η, η'.

4 Consequences of the solution

At first glance this result looks like it may be nothing more than an existance proof, i.e. solutions exist. However, the form of the matrix for after the beamline places some restrictions on the types of beamlines to be used. For example, the chicane type solution is particularly attractive for its simplicity. However, for any beamline that will return the incoming dispersion and its slope to zero with the cavity off, like the chicane, perfect emittance exchange cannot occur. To zero the dispersion and slope without the cavity, assuming no dispersion prior to the beamline, the following conditions must be met:

$$\begin{aligned} D &= -\left(e\eta + f\eta'\right) \\ D' &= -\left(g\eta + h\eta'\right) \end{aligned}$$

These differ from Equations 12 by a negative sign, so the only solution is if $\eta = \eta' = 0$. This also requires an infinite strength deflecting mode cavity.

Most of the designs with which we are familar are based on a dogleg which generates dispersion with zero slope prior to the cavity.

$$M_{bc} = \begin{pmatrix} 1 & L & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & \eta & 1 & \xi \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{15}$$

Pursuing this design leads to three types of solutions. The first is a beamline after the cavity which generates further dispersion without slope.

$$M_{ac} = \begin{pmatrix} \frac{D}{\eta} & f & 0 & D \\ 0 & \frac{\eta}{D} & 0 & 0 \\ 0 & \eta & 1 & \chi \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{16}$$

Setting $D = \eta$ and $f = L$ gives the solution that is proposed in Reference [2]. That is, a double dogleg with a deflecting cavity between the doglegs.

Another solution is to have a beamline which generates zero dispersion but with a slope.

$$M_{ac} = \begin{pmatrix} 0 & \frac{-\eta}{D'} & 0 & 0 \\ \frac{D'}{\eta} & h & 0 & D' \\ 0 & \eta & 1 & \chi \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{17}$$

A beamline composed of a drift-quad-drift-dipole can satisfy the requirements of this matrix if the focal length of the quad is equal to the length of the following drift. This solution is similar to the design that Helen has arrived at [3].

Other beamline designs are also possible which are a composition of the two beamlines where neither the dispersion or its slope are zeroed as some point.

5 MultiCell cavity Matrix

Don as derived in his note the matrix for a single cell deflecting mode cavity[4].

$$M_{cell} = \begin{pmatrix} 1 & L_{cell} & \frac{k_{cell}L_{cell}}{2} & 0 \\ 0 & 1 & k_{cell} & 0 \\ 0 & 0 & 1 & 0 \\ k_{cell} & \frac{k_{cell}L_{cell}}{2} & \frac{k_{cell}^2 L_{cell}}{4} & 1 \end{pmatrix} \tag{18}$$

If we consider a cavity of n cells, each with a strength $k_{cell} = \frac{k}{n}$ and length $L_{cell} = \frac{L}{n}$, where k and L are the total integrated strength and length of the cavity, the cavity matrix becomes

$$M_{cav} = \begin{pmatrix} 1 & \frac{L}{n} & \frac{kL}{2n^2} & 0 \\ 0 & 1 & \frac{k}{n} & 0 \\ 0 & 0 & 1 & 0 \\ \frac{k}{n} & \frac{kL}{2n^2} & \frac{k^2 L}{4n^3} & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & L & \frac{kL}{2} & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & \frac{kL}{2} & \frac{1+2n^2}{12n^2}k^2 L & 1 \end{pmatrix}. \tag{19}$$

In the case of 5 cells, this reduces to

$$M_{cav} = \begin{pmatrix} 1 & L & \frac{kL}{2} & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & \frac{kL}{2} & \frac{17k^2 L}{100} & 1 \end{pmatrix} \tag{20}$$

as was published in Tim's PAC07 paper [5]. Interestingly enough, for a fixed cavity length and strength, as the number of cells goes to infinity (i.e. the high frequency limit) the cavity matrix becomes

$$M_{cav} = \begin{pmatrix} 1 & L & \frac{kL}{2} & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & \frac{kL}{2} & \frac{k^2L}{6} & 1 \end{pmatrix} \tag{21}$$

which is the Cornaccia and Emma long cavity matrix. [1]

6 Cavity Strength

Recent measurements on Tim's cavity suggest that it may not reach the required k value. The question then arises, how do the final emittances vary with the cavity strength. To answer this, I will assume that the beamline has been designed such that Equations 12b and 12c are satisfied, as is the case at the photoinjector. I will use the long 5 cell cavity matrix.

$$M_{cav} = \begin{pmatrix} 1 & L & \frac{kL}{2} & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & \frac{kL}{2} & \frac{17k^2L}{100} & 1 \end{pmatrix} \tag{22}$$

Using Equation 18 does not alter Equations 12 in any way. The major difference however is that $\lambda^2\epsilon_{x0}\epsilon_{z0}$ will never be zero, a fact that was first illustrated by Cornaccia and Emma [1]. Equations 12 still provide the minimium coupling between the planes.

Nevertheless, there are five quantities that determine the final emittances after the exchange. As a function of k, these quantities are:

$$|\mathbf{A}| = 1 + 2k\eta + k^2\left(\eta^2 - \frac{2L^2\eta'^2}{25}\right) \tag{23a}$$

$$\mathbf{A}_{21}\mathbf{B}_{11} - \mathbf{A}_{11}\mathbf{B}_{21} = -k\left(a + \frac{cL}{2}\right) + k^2\left[-a\eta - \frac{33L}{100}\left(c\eta - a\eta'\right) - \frac{2}{25}cL^2\eta\right] \tag{23b}$$

$$\begin{aligned} \mathbf{A}_{21}\mathbf{B}_{12} - \mathbf{A}_{11}\mathbf{B}_{22} &= 2\left(c\eta - a\eta'\right) - cL\eta' + k\left[\left(3\eta - \frac{L\eta'}{2}\right)\left(c\eta - a\eta'\right) - \xi\left(a + \frac{cL}{2}\right)\right] \\ &+ k^2\left[\left(\eta^2 - \frac{2}{25}L^2\eta'^2 - \frac{33}{100}L\xi\right)\left(c\eta - a\eta'\right) - \xi\left(a\eta - \frac{2}{25}cL^2\eta'\right)\right] \end{aligned} \tag{23c}$$

$$\mathbf{A}_{22}\mathbf{B}_{11} - \mathbf{A}_{12}\mathbf{B}_{21} = k\left(b + \frac{dL}{2}\right) + k^2\left[-b\eta - \frac{33L}{100}\left(d\eta - b\eta'\right) + \frac{2}{25}dL^2\eta'\right] \tag{23d}$$

$$\begin{aligned} \mathbf{A}_{22}\mathbf{B}_{12} - \mathbf{A}_{12}\mathbf{B}_{22} &= 2\left(d\eta - b\eta'\right) - dL\eta' + k\left[\left(3\eta - \frac{L\eta'}{2}\right)\left(d\eta - b\eta'\right) - \xi\left(b + \frac{dL}{2}\right)\right] \\ &+ k^2\left[\left(\eta^2 - \frac{2}{25}L^2\eta'^2 - \frac{33}{100}L\xi\right)\left(d\eta - b\eta'\right) - \xi\left(b\eta - \frac{2}{25}dL^2\eta'\right)\right] \end{aligned} \tag{23e}$$

These quantities appear in various combinations in Equations 6 and 10 and and each shows a roughly parabolic dependance of the exchanged emittances as a function of cavity strength. I say roughly because these quantities appear in combinations under a radical, so the dependance is really a square root of a quartic equation. It is also interesting to note that equations 23d and 23e can be obtained by substituting $c \to d$ and $a \to b$ in equations 23b and 23c.

For illustration and to aid future discussion, the $L = 0$ limits of these quantities are

$$\begin{aligned}
|\mathbf{A}| &= (1+k\eta)^2 && \text{(24a)}\\
\mathbf{A}_{21}\mathbf{B}_{11} - \mathbf{A}_{11}\mathbf{B}_{21} &= -ak\,(1+k\eta) && \text{(24b)}\\
\mathbf{A}_{21}\mathbf{B}_{12} - \mathbf{A}_{11}\mathbf{B}_{22} &= 2\,(c\eta - a\eta') + k\,[3\eta\,(c\eta - a\eta') - a\xi] &&\\
&\quad + k^2\eta\,[\eta\,(c\eta - a\eta') - a\xi] && \text{(24c)}\\
\mathbf{A}_{22}\mathbf{B}_{11} - \mathbf{A}_{12}\mathbf{B}_{21} &= -bk\,(1+k\eta) && \text{(24d)}\\
\mathbf{A}_{22}\mathbf{B}_{12} - \mathbf{A}_{12}\mathbf{B}_{22} &= 2\,(d\eta - b\eta') + k\,[3\eta\,(d\eta - b\eta') - b\xi] &&\\
&\quad + k^2\eta\,[\eta\,(d\eta - b\eta') - b\xi] && \text{(24e)}
\end{aligned}$$

For $k = -\frac{1}{\eta}$, these quantities become

$$\begin{aligned}
|\mathbf{A}| &= \frac{2}{25}\left(L\frac{\eta'}{\eta}\right)^2 && \text{(25a)}\\
\mathbf{A}_{21}\mathbf{B}_{11} - \mathbf{A}_{11}\mathbf{B}_{21} &= \frac{L}{\eta}\left(\frac{17}{100}c + \frac{33}{100}\frac{a\eta'}{\eta}\right) + \frac{2}{25}\frac{cL^2\eta'}{\eta^2} && \text{(25b)}\\
\mathbf{A}_{21}\mathbf{B}_{12} - \mathbf{A}_{11}\mathbf{B}_{22} &= \frac{L}{\eta}\left[-\eta'\,(c\eta + a\eta') + \frac{17}{100}c\xi + \frac{33}{100}\frac{\eta'}{\eta}a\xi\right] &&\\
&\quad + \frac{L^2}{\eta^2}\left[-\frac{2}{25}\eta'^2\,(c\eta + a\eta') + \frac{2}{25}c\eta'\xi\right] && \text{(25c)}\\
\mathbf{A}_{22}\mathbf{B}_{11} - \mathbf{A}_{12}\mathbf{B}_{21} &= \frac{L}{\eta}\left(\frac{17}{100}d + \frac{33}{100}\frac{b\eta'}{\eta}\right) + \frac{2}{25}\frac{dL^2\eta'}{\eta^2} && \text{(25d)}\\
\mathbf{A}_{22}\mathbf{B}_{12} - \mathbf{A}_{12}\mathbf{B}_{22} &= \frac{L}{\eta}\left[-\eta'\,(d\eta + b\eta') + \frac{17}{100}d\xi + \frac{33}{100}\frac{\eta'}{\eta}b\xi\right] &&\\
&\quad + \frac{L^2}{\eta^2}\left[-\frac{2}{25}\eta'^2\,(d\eta + b\eta') + \frac{2}{25}d\eta'\xi\right] && \text{(25e)}
\end{aligned}$$

all of which reduce to zero for a zero length cavity. These equations also serve as a starting point to optimize the beamline or the beam parameters to minimize the residual emittance coupling.

To illustrate how these parameters affect the emittances of the beam as a function of cavity strength, I have used ELEGANT to simulate the beam emittances after emittance exchange line installed in A0. The model does not include effects like space charge or CSR, and the cavity is treated as the above matrix.

Figure 6 shows how $|A|$ changes with cavity strength. Figure 6 shows how the other constants vary with cavity strength. The magnitudes of all of the parameters area minimum

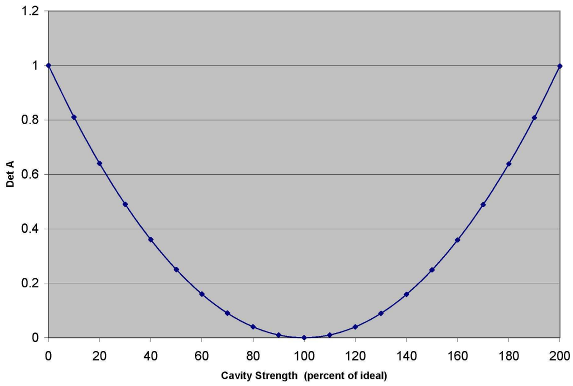

Figure 1: Det A vs. cavity strength. The horizontal axis is the percent of ideal strength.

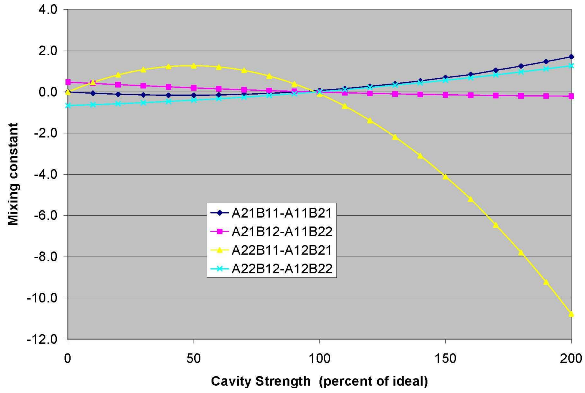

Figure 2: Mixing Constants vs. cavity strength. The horizontal axis is the percent of ideal strength.

when the cavity is at the ideal strength and grow in either direction. From these graphs alone it is hard to see how the emittances themselves change.

Figure 6 shows how the emittances after the emittance exchange line vary with the cavity strength. For this graph, $\epsilon_x = 5\text{mm} - \text{mrad}$ and $\epsilon_z = 110\text{mm} - \text{mrad}$ at the start of the exchange line. The blue trace is the initial longitudinal emittance normalized to

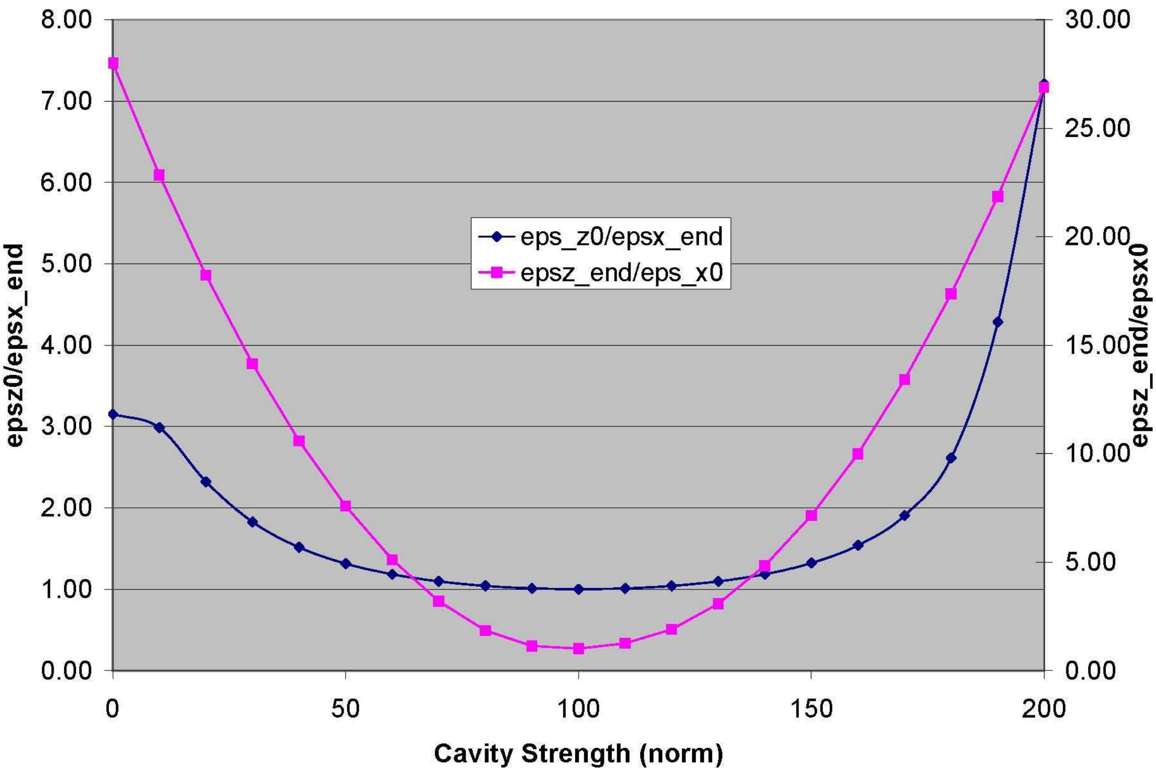

Figure 3: Transverse and Longitudinal emittance growth vs. cavity strength. The horizontal axis is the percent of ideal strength.

the final transverse emittance and is plotted on the left axis. The red trace is the final longitudinal emittance normalized to the inital transverse emittance. I should note that no attempt was make to optimize the beam parameters. What the figure shows is that the final longitudinal emittance is much more sensitive to changes in cavity gradient than the transverse. If the cavity strength is 70% of ideal, the transverse emittance only grows 10%, while the longitudinal emittance grows 320%. Considering the sensitivity of the longitudinal emittance to the cavity strength, it is worthwhile investing time to optimize the input beam parameters to be more insensitve to this.

Figure 6 shows how the emittances after the emittance exchange line vary with the cavity strength. For this graph, $\epsilon_x = 5\text{mm} - \text{mrad}$ and $\epsilon_z = 110\text{mm} - \text{mrad}$ at the start of the exchange line. The blue trace is the initial longitudinal emittance normalized to the final transverse emittance and is plotted on the left axis. The red trace is the final longitudinal emittance normalized to the inital transverse emittance. I should note that no attempt was make to optimize the beam parameters. What the figure shows is that the final longitudinal emittance is much more sensitive to changes in cavity gradient than the transverse. If the cavity strength is 70% of ideal, the transverse emittance only grows 10%, while the longitudinal emittance grows 320%. Considering the sensitivity of the longitudinal emittance to the cavity strength, it is worthwhile investing time to optimize the input beam parameters to be more insensitve to this.

7 Conclusion

The design requirements of a transverse to longitudinal emittance exchange beamline have been expanded from the initial description given in Reference [1]. Using only the assumptions of symplecticity and no accelerating RF we have been able to show the properties needed for the beamlines before and after the deflecting mode cavity. The beamline prior to the cavity only has to generate the desired dispersion at the cavity. The cavity strength has to be matched to the generated dispersion. The beamline after the cavity needs to satisfy Equations 12 to effect a perfect emittance exchange.

If the beamline prior to the cavity generates dispersion with no slope, three types of solutions appear for after the cavity. The first is a solution which generates further dispersion with no slope, such as the dogleg proposed by Reference [2]. The second causes the dispersion to go through zero at some point, such as the solution proposed by Reference [3]. The last is an admixture of the two.

A final consequence is that standard chicane type solutions will not produce a perfect emittance exchange.

The cavity matrix for a multicell deflecting mode cavity has been analyzed, deriving the genreal case of an n cell cavity. In the limit of an inifinite number of cell, the Cornaccia and Emma long caivty matrix is derived.

A preliminary sensitivity analysis of the cavity strength has been performed. It shows that the longitudinal emittance after for exchange is sensitive to the cavity strength, while the transverse emittance after the exchange is not.

References

[1] M. Cornacchia and P.Emma. Phys. Rev. ST Accel. Beams 5, 084001 (2002).

[2] P. Emma, Z. Huang, K.-J. Kim,and P. Piot. Phys. Rev. ST Accel. Beams 9, 100702 (2006).

[3] Helen Edwards. Private Communication.

[4] D.A. Edwards. Unpublished.

[5] T. Koeth, L. Bellantoni, D. Edwards, H. Edwards, R. P. Fliller III. ”A TM110 Cavity for Longitudinal to Transverse Emittance Exchange”, Proceedings of the 2007 Particle Accelerator Conference, THPAS079.